Addressing forest ecosystem management planning concerns with linear programming. An application in Portugal

Olha Nahorna

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Supervisor: José Guilherme Martins Dias Calvão Borges

Jury:

President: PhD Pedro César Ochôa de Carvalho, Assistant Professor at Instituto Superior de Agronomia, Universidade de Lisboa.

Members: PhD José Guilherme Martins Dias Calvão Borges, Associate Professor with Habilitation at Instituto Superior de Agronomia, Universidade de Lisboa;

PhD Susete Maria Gonçalves Marques, Researcher at Instituto Superior de Agronomia, Universidade de Lisboa.

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Abstract

Forest management is an extremely complex process that requires the combination of various techniques, practices and methods in order to achieve given environmental, economic and social objectives. Linear programming (LP) is one of the most widely used optimization methods that assists forest managers in the process of the decision-making. The use of alternative formulations of the LP model may help acquire insights about the forest ecosystem management planning problem, may thus lead to better plans.

This work presents the study of influence of different LP model formulation on the design of the management plan and on economic values, timber flow, tree species distribution, total carbon stock, cork extracted, biodiversity and cultural services. A total of 16 model formulations (scenarios) were considered for the analysis. Scenarios were obtained by changing the objective function and by adding of management related constraints. The set of objective functions included the minimization of costs, the maximization of the net present value (NPV) over the planning horizon, the ending inventory value (EIV), and the total present value (PVFI = NPV + EIV). The set of constraints included 10% timber even-flow constraints and timber targets per period and per tree species. The study area was Vale do Sousa, Portugal.

The results of the study demonstrated that the LP model formulation has a substantial influence on the proposal of management plan. It allowed to check the trade-offs between economic criteria and changes in timber flows, tree species distribution, extracted cork and carbon stock. Biodiversity and cultural services remained at the same level across scenarios. Results suggest the importance of using alternative formulations to acquire information about the management plan and to explore responses to alternative scenarios and to make better decisions.

Keywords: linear programming, forest management, ecosystem services, forest ecosystem values, optimization methods.

Resumo

O pleneamento da gestão florestal é um processo extremamente complexo que requer a combinação de várias técnicas, práticas e métodos para atingir determinados objetivos ambientais, económicos e sociais. A programação linear (PL) é um dos métodos de otimização mais amplamente utilizado que auxilia os gestores florestais no processo de tomada de decisão. No entanto, é crucial entender que diferentes formulações do modelo PL podem levar a resultados substancialmente diferentes.

Este trabalho apresenta o estudo da influência de diferentes formulações de modelos de PL na definição do plano de gestão florestal. Estas formulações consideram critérios económicos, o fluxo de madeira, a distribuição de espécies arbóreas, carbono armazenado, cortiça extraída, biodiversidade e serviços culturais. O estudo envolveu um número total de 16 formulações (cenários). Especificamente, os cenários foram obtidos alterando a função objetivo e adicionando restrições a objetivos de oferta de serviços de ecossistema. As funções objetivo incluíram a minimização de custos, a maximização do valor atual líquido (VAL) ao longo do horizonte de planeamento, a maximização do valor do inventário final (EIV) e o valor atual liquido total (PVFI = VAL + EIV). As formulações envolveram restrições relativas ao fluxo de madeira ao longo do horizonte de planeamento (máximo de 10% entre períodos) e os valores de oferta de madeira de cada espécies florestal em cada período. A área de estudo foram Zonas de Intervenção Florestal (ZIF's) Entre Douro e Sousa e Castelo de Paiva no Vale do Sousa, Portugal.

Os resultados do estudo demonstraram que a formulação do modelo PL tem grande influência substancial na proposta do plano de gestão. Estas permitiram observar as alterações nos parâmetros económicos estudados bem como nos fluxos de madeira, distribuição de espécies arbóreas, cortiça extraída e carbono armazenado. Os valores de biodiversidade e de serviços culturais não sofreram alterações substanciais. Os resultados sugerem a importância de utilização da PL e de formulações alternativas para adquirir informação sobre o problema de planeamento da gestão florestal, para explorar cenários alternativos e para definir melhores planos de gestão florestal.

Palavras-chave: programação linear, gestão florestal, serviços de ecossistema, valores dos ecossistemas florestais, métodos de otimização.

Resumo alargado

O planeamento da gestão florestal é um processo extremamente complexo que requer a combinação de várias técnicas, práticas e métodos para atingir determinados objetivos ambientais, económicos e sociais. A programação linear (PL) é um dos métodos de otimização mais amplamente utilizados e que auxilia os gestores florestais no processo de tomada de decisão. Normalmente, é aplicado para resolver o problema de como afetar recursos limitados entre atividades concorrentes por forma a satisfazer os objetivos do planeamento da gestão. Entretanto, é importante utilizar a PL como instrumento para produzir mais informação sobre o problema de planeamento da gestão.

O objetivo deste trabalho foi estudar a influência de diferentes formulações do modelo LP na definição do plano de gestão florestal. Estas formulações consideram critérios económicos, o fluxo de madeira, a distribuição de espécies arbóreas, carbono armazenado, cortiça extraída, biodiversidade e serviços culturais. As áreas de estudo foram as Zonas de Intervenção Florestal (ZIF) Paiva e de Entre-Douro e Sousa (ZIF_VS), localizadas no noroeste de Portugal, a aproximadamente 50 Km da cidade do Porto. A área total do Vale do Sousa estende-se por 14,760 ha e foi classificada em 1,343 unidades de gestão. Os serviços de ecossistema fornecidos pelo ZIF_VS incluem rolaria de eucalipto e madeira para serração de pinheiro bravo, bem como armazenamento de carbono. Um total de 250,100 prescrições foram simuladas para o conjunto de unidades de gestão (povoamentos) considerando um horizonte de planeamento de 90 anos, com nove períodos de 10 anos. Estas prescrições foram classificadas em oito programas de gestão florestal: povoamento misto com pinheiro bravo (*Pinus pinaster* Ait.) e eucalipto (*Eucalyptus globulus* Labill) com dominância do pinheiro (Programa 1); Programa 2 - povoamento misto com pinheiro bravo e eucalipto com dominância de eucalipto; – povoamento puro de castanheiro (*Castanea sativa* Mill.) (Programa 3); povoamento puro de eucalipto (Programa 4); povoamento puro de pinheiro bravo (Programa 5); povoamento puro de carvalho roble (*Quercus robur* L.) (Programa 6); povoamento puro de sobreiro (*Quercus suber* L.) (Programa 7); povoamento com espécies ripícolas (Programa 8).

Considerou-se um modelo de programação linear de tipo Modelo I para representar o problema de planeamento da gestão. Com base nele, desenvolveram-se 16 formulações diferentes (cenários) para a análise do problema de decisão. Os cenários foram obtidos alterando a função objetivo e adicionando restrições relativas ao valor de oferta de serviços de ecossistema. As funções objetivo incluíram a minimização de custos, a maximização do valor atual líquido (VAL) ao longo do horizonte de planeamento, a maximização do valor do inventário final (EIV) e a maximização do valor atual liquido total (PVFI = VAL + EIV). As formulações envolveram restrições relativas ao fluxo de madeira ao longo do horizonte de planeamento (flutuação máxima de 10% entre períodos) e aos valores de oferta de madeira de cada espécie florestal em cada período. As metas obrigam a que o volume realizado em cada período seja superior ou igual a 1,000,000 m^3. Para além disso, as metas obrigam a valores mínimos de volume realizado ao longo do horizonte de planeamento para cada espécie, ex., 3,000,000 m^3 no caso do pinheiro bravo, 6,000,000 m^3 no caso do eucalipto 200,000 m^3 no caso do castanheiro 10,500 m^3 no caso do carvalho e 250,000 m^3 no caso do sobreiro.Todos os modelos foram resolvidos com o software CPLEX e analisados separadamente, com posterior comparação entre cenários, a fim de identificar correlações e dependências.

Para obter o valor do inventário final, para cada combinação de unidade de gestão e prescrição, foram desenvolvidas as seguintes etapas: i. Foi calculado o valor no ano 90 de todas as receitas e custos ocorridos desde o final do horizonte de planeamento até o final da rotação em curso; ii. O Valor Esperado do Solo (SEV) foi obtido e descontado do ano do final dessa rotação para o ano 90; iii. A soma dos valores calculados em i. e ii. foram atualizados tomando como referência o ano inicial do horizonte de planeamento. No caso do sobreiro, o seu valor do inventário final foi estimado como o valor de uma série perpétua de extrações de cortiça.

Os valores mais elevados das funções objetivo, como esperado, foram observados nos cenários sem restrições adicionais aplicadas. Os segundos melhores valores da função objetivo, no caso dos cenários de maximização de NPV, EIV e PVFI, foram observados em cenários em que se incluem as restrições de madeira alvo por volume a realizar por espécie são aplicadas. No caso de minimização de custos, o segundo melhor resultado foi observado no cenário, onde foram incluídas as restrições de fluxo de madeira de 10%.

Em todos os casos, as espécies florestais mais utilizadas foram o pinheiro bravo e o eucalipto, seguidos pelo castanheiro e sobreiro. Isto reflete o inventário atual. As prescrições de carvalho foram escolhidas apenas quando restrições correspondentes foram aplicadas. As espécies ribeirinhas sempre receberam o número mínimo de hectares, conforme determinado pelas restrições de área correspondentes. De acordo com o valor do EIVs em todos os cenários, a maior parte do inventário final está concentrada em áreas de pinheiro bravo, seguidas de eucalipto. As prescrições de sobreiros e castanheiros trazem uma contribuição muito menor. O carvalho, mesmo quando suas prescrições são incluídas, contribui muito pouco ou até negativamente. Isto reflete as condicionantes que decorrem do inventário atual, da possibilidade de reconversões e dos benefícios e custos associados aos modelos de silvicultura.

A análise dos preços-sombra que correspondem às restrições de área mostrou que as áreas de pinheiro bravo e eucalipto têm maior valor principalmente nos casos em que a função objetivo era maximizar o VPL, minimizar os custos ou maximizar o PVFI. Os povoamentos de sobreiro são muito importantes quando a função objetivo é maximizar o EIV ou PVFI. Finalmente, os povoamentos de carvalho e de castanheiro tornam-se mais importantes quando são introduzidas restrições de madeira por espécie.

O fluxo de madeira altera-se com a introdução de restrições de gestão. Em cenários em que nenhuma restrição é incluída observam-se picos de volume a realizar muito altos determinados períodos, com escassez significativa em outros. Restrições de fluxo de madeira com flutuação máxima de 10% entre períodos e relativas a metas de madeira por período permitem obter um rendimento de madeira mais regular ao longo do horizonte de planeamento. No entanto, em todos os casos, observa-se algum aumento de volume a realizar no período 2 devido à distribuição etária dos povoamentos no inventário inicial. Além disso, há um aumento substancial no Período 5 e o pico mais alto no Período 6, devido a acumulação de madeira de novas áreas plantadas durante os primeiros períodos, seguido de uma diminuição significativa até ao Período 9, quando ocorre a próxima grande acumulação de volume. A análise dos preços-sombra que correspondem a restrições de fluxo de madeira com flutuação máxima de 10% entre período e as metas de madeira por período comprovaram essa informação.

O carbono armazenado muda substancialmente entre cenários. Normalmente é mais baixo em caso de problemas de minimização de custos e de maximização de EIV. A introdução de restrições de fluxo de madeira com flutuação máxima de 10% entre períodos leva ao aumento de carbono armazenado. A introdução de restrições de madeira por período e por espécie também leva ao aumento de carbono armazenado, muito maior em caso de maximização da EIV ou de minimização de custos, do que restrições de fluxo com flutuação máxima. Ao mesmo tempo, a biodiversidade e os serviços culturais permaneceram no mesmo nível em todos os cenários. A quantidade de cortiça extraída depende do número de hectares atribuídos às prescrições de sobreiro.

Os resultados do estudo demonstraram que a formulação do modelo LP tem influência substancial na proposta do plano de gestão. Permitiram observar as alterações nos parâmetros económicos estudados bem como nos fluxos de madeira, distribuição de espécies arbóreas, cortiça extraída e carbono armazenado. Os valores de biodiversidade e de serviços culturais não sofreram alterações substanciais. Os resultados sugerem a importância de utilização da PL e de formulações alternativas para adquirir informação sobre o problema de planeamento da gestão florestal, para explorar cenários alternativos e para definir melhores planos de gestão florestal.

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AFI – Area for Forest Intervention

CPF – Corner-Point Feasible

DSS – Decision Support System

EIV – Ending Inventory Value

KS – Knowledge System

LP – Linear Programming

LS – Language System

NPV – Net Present Value

PPS – Problem Processing System

PS – Presentation System

PVFI – Present Value of all Future Incomes

SEV – Soil Expectation Value

ZIF – Zona de Intervenção Florestal

1. INTRODUCTION

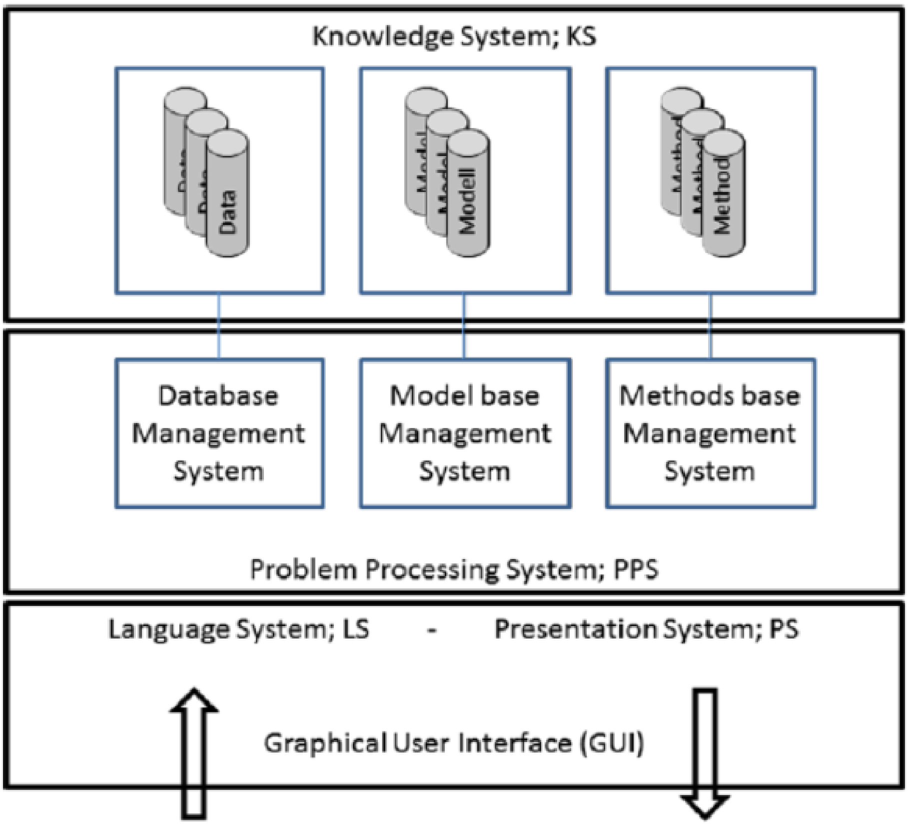
1.1. WORK SCOPE

Forest management is a complex process of planning and application of appropriate techniques, principles and practices in order to achieve specific environmental, economic and social objectives. The planning process includes identification of forest owner's objectives, resources inventory, development and implementation of the management plan with further periodic evaluation and adjustments if needed (Davis et al., 2001).

Forests are characterized by their multiple-use, such as supply of timber and non-wood forest products, water protection, wildlife, clean air, recreation and aesthetics (Sabogal et al., 2014). Thus, in order to achieve responsible and effective forest management, managers need to have a clear understanding of how does the whole ecosystem work. Moreover, managers have to take into account different time scales; given that, forests take long time to regenerate and therefore each management decision should be made with clear understanding of what are the not only short-term, but also long-term consequences. Spatial scale is also an issue, as managers may have to deal with large forest areas. Therefore, given all the complexity of forest management process, various computerized decision support systems (DSS) have been developed and are being actively used by forest managers (Eriksson & Borges, 2014).

Decision support system, in a broad sense, can be seen as an interacting summation of anything that could help the decision-maker. In a computer science context, DSS is a model-based software system, which includes four main components: (i) a language system (LS) that allows users to interact with DSS, (ii) a presentation system (PS) that displays the results, (iii) a knowledge system (KS) that stores all the input information and (iv) a problem processing system (PPS) (Burstein & Holsapple, 2008).

Language, presentation and knowledge systems are representative systems: LS includes all the messages to the DSS from the user; PS includes all the messages to the user from the DSS; KS includes all the knowledge stored in the form of data or models in the DSS. In turn, KS is divided into three sub-components: (i) one that stores the data, (ii) one that stores the models, (iii) one that stores the methods to be used. Problem processing system is the integrative component, which solves the problem specified by the user. It receives the command from the LS, integrates the data, models and methods from the KS and represents the output to the PS (Figure 1).



Source: Eriksson & Borges, 2014

Figure 1. Schematic diagram of the main components of a DSS.

Let us consider the components of the KS. Models are abstract representation of the reality. In forestry, models help to understand the dynamics of forests, evaluate management alternatives and their impacts, study the evolution of different tree species and other vegetation (Mendoza & Vanclay, 2008). There are various classifications of the models, among them there are simulation and optimization models.

Simulation models are used for practical implementation of growth and yield models to enable projections of the future timber production (Twery, 2014). Forest simulators are special tools that can predict the production of wood and non-wood forest products at any point of the time, at different spatial scale (Tomé & Faias, 2011). This could allow managers to perform long-term forecasting of the state of forest or forest area under various management alternatives and climate scenarios. However, it is crucial to understand that simulations support decision-making by evaluation of given options, but they do not provide the optimal strategy to follow (Lee, 2015).

Optimization models usually deal with large datasets and a big number of calculations. The main problem that optimization models address is which treatment, at which stand and at what time should be applied in order to achieve given objectives, while meeting all predefined organizational and environmental constraints (McDill, 2014).

There are various optimization methods that are widely implemented to address forest management, among them: classical or exact methods (linear programming, goal programming, dynamic programming) (Garcia, 1990), heuristic or probability-based methods (Sessions et al., 2007), multiple criteria decision analysis (Mendoza & Martins, 2006).

The work of this dissertation was focused on linear programming optimization method. The study area was Vale do Sousa, which is located approximately 50 km east of the city of Porto, in North-Western Portugal region. The updated linear programming model version was developed based on previously reported studies (Borges et al., 2017; Borges et al., 2014a; Marques et al., 2017).

1.2. OBJECTIVES

The main objective of this dissertation is to study the forest by analysing the influence of changes in linear programming model formulation on the final output.

Specific research objectives include:

- To calculate values of ending inventory for each combination of prescription and management unit.
- To create equations that will compute the total value of ending inventory and update the LP model with them.
- To run the model with different objective function and compare the results.
- To add additional constraints to the model and analyse the changes in the output.

1.3. STRUCTURE

The dissertation consists of six main chapters. A brief description of content of each chapter is provided below:

1. **Introduction**: presents the general description of the research background, research problem and objectives.

2. **Background**: presents the literature review of the general concept of linear programming and its uses in forestry. The chapter introduces main definitions, characteristics, formulation and interpretation of LP models.

3. **Materials and Methods**: presents the study area and its characteristics, data and its structure, formulation of the model and its description.

4. **Results**: presents the output from model runs.

5. **Discussion**: summarises the research results and presents the key findings of the thesis.

6. **Conclusions:** presents the importance of the study, limitations, implication of the results and highlights the paths for the future research.

2. BACKGROUND

2.1. LINEAR PROGRAMMING

Linear programming (LP) is a mathematical optimization technique, in which the objective function and the constraints are expressed as system of linear functions of the decision variables (McDill, 2014). Typically, it is used to address problem of allocating some type of limited resources among competing activities in an optimal way to satisfy given objectives.

Linear programming was developed independently by the Soviet economist Leonid Kantorovich (1939) and by the Dutch economist Tjalling Koopmans. However, the American mathematician George Dantzig had introduced the general formulation of linear programming in 1947. Initially it was developed for the planning activities of the U.S. Air Force. Shortly after that, LP came into a wide use in many different fields, including petroleum industry, food-processing industry, metalworking industries, as well as financial management and natural resources management (Dantzig & Thapa, 2006).

2.1.1. Characteristics of linear programming technique

Typical LP model consists of the objective function, one or more constraints and possibly few accounting rows. There are two key elements of LP models: (1) all the relationships between elements of the model must be quantifiable, and (2) all the relationships must be expressed linearly (Bettinger et al., 2009). Thus, the output of the model provides also quantitative assessment of the given management options.

In order to formulate linear programming model, it is necessary to take into account four main assumptions of proportionality, additivity, divisibility and certainty (Bettinger et al., 2009). According to the assumption of *proportionality*, the contribution of each decision variable to the objective function is proportional to the value of this decision variable. Assumption of *additivit*y states that each decision variable contribute to the objective function independently from other variables. *Divisibility* assumes that all decision variables are continuous real numbers; however, this may not be applicable to some real life problems, when solution has to be an integer number. In this case mixed integer programming or integer programming are applied (Buongiorno & Gilless, 2003). Finally, the last assumption in linear programming is assumption of *certainty*, thus, it considers that all given parameters are known with certainty.

In case of linear programming, the term *solution* does not mean the final answer; it corresponds to any specification of values of decision variables (Hillier & Lieberman, 2010). A *feasible solution* is a solution obtained when all the constraints are satisfied. An *infeasible solution* is obtained when at least one constrained is violated. Thus, the *feasible region* is the region that includes all the feasible solutions of the problem. The LP model is aiming to obtain an *optimal solution*, which is a feasible solution that corresponds to the most favourable value of the objective function (the largest possible value in case of maximization problem and the smallest value in case of minimization problem). Another important term is a *corner-point feasible (CPF) solution*, which is a solution that lies at the corner of the feasible region (Hillier & Lieberman, 2010). It can be also referred as an *extreme point* or a *vertex*. CPF solution plays a key role when applying the *Simplex* method.

George Dantzig introduced the Simplex method in 1947 together with the general formulation of linear programming. The Simplex method solves the linear programming problem by moving along the boundaries from one extreme point to the next and improving the value of the objective function with each move (Bradley et al., 1977). It is an efficient method for solving large LP models and it is applied in the most of software. The Simplex method goes through two-phase process. In the first phase, it obtains an initial feasible solution, if one exists. In the second phase, it obtains an optimal solution (Dantzig & Thapa, 2006).

Although the Simplex method is widely used, there are other available methods and algorithms. However, in fact, some of them are just variants of the simplex method itself. For instance, the dual simplex method. It is based on the duality theory, which says that any primal linear programming problem can be converted into a dual problem. The dual problem is obtained by transforming each decision variable into a constraint, each constraint into a decision variable and inversing the objective function (Dantzig & Thapa, 2006). Thus, the dual simplex method deals with a primal basic solution that is dual feasible and then moves towards an optimal solution by striving to achieve primal feasible solution as well (Hillier & Lieberman, 2010). Other examples of available methods are parametric linear programming, upper bound technique, interior-point approach.

2.1.2. Linear programming software

First linear programming models were solved with a pencil, a piece of paper and a table slide rule. This could allow only little models to be developed and could take a lot of time to complete all the calculations. Nevertheless, the technological progress does not slow down and nowadays much harder and bigger LP problems can be solved with the use of special software.

There are various programs designed to help users to develop and solve linear programming models. In this section, we will consider few of them.

Some LP problems can be solved with the Excel spreadsheets that includes special build-in optimizer Solver. Buongiorno and Gilless (2003) provide various examples of practical use of Excel Solver tool for linear programming models. However, it would not be applicable for big complex problems.

One of the most widely used tools for building and solving linear programming models is LINGO (Language for Interactive General Optimizer) (e.g. Demirci & Bettinger, 2015; Keleș & Bașkent, 2011). It allows creating mathematical optimization models by taking needed information directly from spreadsheets and databases and similarly storing model output back to the spreadsheet or database (LINDO Systems, 2017). However, certain versions of the software have limitations for optimizable variables and constraints.

Another one widely used software is CPLEX (Campanella et al., 2018; Fuentealba et al., 2019). It was the first commercial optimizer that was written in the C programming language. Its name derives from the concept of Simplex algorithm being written in C language, thus C-Simplex became CPLEX. It is characterized by high performance level, as it allows to solve large problems with big amount of decision variables and constraints with a fast data processing (IBM Corp., 2020). The CPLEX Optimizer directly reads the problem from the file saved in certain standard formats, solves the problem and returns the solution interactively or to a separate text file (IBM Corp., 2017).

2.2. LINEAR PROGRAMMING APPLIED TO FORESTRY

In forestry, linear programming is normally used to allocate resources and activities across the forest area over a long period. It also allows to understand the trade-offs between management objectives and among management constraints (Bettinger et al., 2009). Firstly, only small problems could have been solved with LP. As one of the main limitation of the technique, Kidd et al. (1966) specified the capabilities of the software and the speed of the computers. However, fast development of new technologies nowadays allow us to solve LP problems with thousands of variables and constraints. Garcia (1990) reviewed a number of classic forest management planning approaches and stated that linear programming is an optimal technique to achieve large computational advantages, as LP codes are extremely effective in handling big datasets. Given that LP is a non-spatial method, there is a big potential in linking with spatial simulation models, which helps managers to evaluate those resource attributes, that are spatially dependent (such as habitat) (Gustafson et al., 2006).

Linear programming is one of the most widely used techniques for forest management worldwide. Most of the U.S. National Forest plans since 1970s have been developed with the use of LP (Kent et al., 1991). It is also applied in Portugal (Borges et al., 2014a; Borges et al., 1997), Ukraine (Nijnik et al., 2012) and many other countries.

2.2.1. Linear programming model formulation

The first and most crucial step in formulation a linear programming model is to identify *decision variables*. They are defined as set of quantities to be found for solving LP problem (McDill, 1999). Normally, the decision variables are expressed as the amount of resource to be managed or the level of certain activity. For example, decision variable can be the number of hectares of forestland to be managed under certain prescription. It can be defined as follows:

x_{ij} – the area of the analysis area *i* to be managed under prescription *j*.

According to Johnson and Scheurman (1977) there are two methods of defining the decision variables, which are Model I and Model II. In a Model I LP problem, the decision variables correspond to all management activities that will be applied to a given stand or strata over the whole planning horizon according to a given prescription. In a Model II formulation, decision variables correspond only to management activities until the final cut. After that, separate regeneration variables are generated.

An *objective function* of a linear programming model is formulated as a linear equation. The model uses it to conduct evaluation of all combinations of management options. The objective function is associated to the notion, which states that it must be either maximized or minimized. For example, the maximization notion can be stated for the income, net present value (NPV) or timber harvested, while minimization notion can be used for costs (Borges et al., 2014b). For example, the general formulation of the objective function that aims to maximize NPV can be presented as follows:

$$Max\ Z = \sum_{i}^{M} \sum_{j}^{N} a_{ij} x_{ij} \tag{2.1}$$

Where:
Z is the value of the objective function;
M is the set of analysis areas;
N is the set of prescriptions for each analysis area i;
a_{ij} is the net present revenue from managing one unit of analysis area i under prescription j.

If we stop formulating the problem here, it is likely that the optimal solution of the LP model will assign all decision variables equal to infinity (in case if all coefficients are positive), which is not feasible. Thus, certain constraints have to be added to the model.

In all forestry problems, the first set of constraints are *area constraints*. They aim to specify amount of area available for each management unit. Then, the general formulation can be gives as follows:

$$\sum_{j}^{N} x_{ij} \leq A_i \tag{2.2}$$

Where A_i is the available area of analysis area i and all the other notions as described before.

Furthermore, other resources constraints can be introduced, for instance available work force, time or equipment. This kind of constraints can be classified as *quantity constraints*. They restrict some part of output values to be equal to, greater than or less than some given value in a period, set of periods or total planning horizon (McDill, 2014). The aim of these constraints is to assure that certain policy restrictions are met and minimum management goals are satisfied.

Finally, the last but not least constraint that must be introduced to any LP model is *non-negativity constraint*, as decision variables normally refer to areas and their values have to be positive. Then:

$$x_{ij} \geq 0 \quad (2.3)$$

Another type of constraints in LP models are *flow constraints*. They restrict some amount of output values in one period to be similar to the same output values in another period. This can be, for instance, imposed rule of non-declining timber harvest, which assures that the amount of harvested timber in a given period must be equal or greater than amount harvested during the previous period (Kent et al., 1991). However, it is not always reasonable to restrict the model for non-declining production of timber. Another example of even-flow constraint can be the restriction that volume harvested in a given period can fluctuate in a range of certain percent from the volume harvested in the previous period (Bettinger et al., 2009). For instance, we know that H_1 and H_2 are amounts of timber harvested in period one and two respectively. We impose the rule, which will allow fluctuation of 10%. Then, the even-flow constraint is:

$$|H_2 - H_1| \leq 0.1H_1 \quad (2.4)$$

or

$$H_2 - 1.1H_1 \leq 0 \quad (2.5)$$

$$H_2 - 0.9H_1 \geq 0 \quad (2.6)$$

LP model could also include some *accounting rows*. They are used in case if there is need to aggregate some values for reporting purposes (Bettinger et al., 2009). For instance, LP model was built with the objective function of maximizing the net present value over a 20-years planning horizon. In the same time, the landowner wants to know the amount of hectares harvested in each 10-year period. We know that X_{11} and X_{21} are decision variables, that represent areas to be harvested during the first period and X_{12} and X_{22} are areas to be harvested in the second period. The accounting variables can be formulated as $AreaHarv_1$ and $AreaHarv_2$ for respectively first and second period:

$$X_{11} + X_{21} = AreaHarv_1 \quad (2.7)$$

$$X_{12} + X_{22} = AreaHarv_2 \quad (2.8)$$

Thus, when the LP model is solved, together with an optimal solution to satisfy the objective function, it will provide the values of accounting variables. Moreover, accounting variables also can be used to formulate new constraints. For instance, we could use given variables

$AreaHarv_1$ and $AreaHarv_2$ to state the minimum or maximum area to be harvested in each period or set the maximum difference between these values by creating flow constraints. In total, it is very useful to introduce accounting variables to big models in order to simplify interpretation of results and further formulation of the model (Kent et al., 1991).

To sum up, the formulation of linear programming model consists of four main steps:

1) choose the decision variables;
2) formulate the objective function;
3) decide and write constraints;
4) write non-negativity constraints.

2.2.2. Linear programming output interpretation

When the LP problem is solved, the solver will return the optimal values of the objective function and the decision variables. Apart from that, the output will also include certain useful values, namely *reduced or opportunity cost*, *slack* or *surplus* and *dual* or *shadow prices* (McDill, 1999).

Reduced or opportunity costs are associated to each decision variable. The reduced cost value is non-zero only for non-basic decision variables, meaning those decision variables that are equal to zero in an optimal solution of the problem. Thus, according to McDill (2014), *reduced cost* value "indicates how much the objective function coefficient on the corresponding variable must be "improved" before the value of the variable will be positive in the optimal solution". In the maximization problems, reduced costs are strictly negative and in the minimization problems, they are always positive (Bradley et al., 1977). Therefore, "improving" variable in the maximization problem means "increasing" and correspondingly "reducing" in the minimization problem. However, if the reduced cost is equal to zero, but the decision variable is still non-basic, this suggests that there may be more possible optimal solutions of the problem (Bradley et al., 1977). The units of the reduced costs are the same as units of the corresponding decision variables coefficients in the objective function.

Slack or surplus values are associated to each constraint. In case of less than or equal constraints, we use the term "slack" and the term "surplus" is used for greater than or equal constraints. The *slack* corresponds to the amount of the resource that was not used and the *surplus* represents the additional extra amount of the resource that was used over the given constraint (McDill, 1999). Therefore, the value of slack or surplus is non-zero only when the corresponding constraint is not binding. The units of slack or surplus are the units of the given constraints.

Shadow or dual prices correspond to each constraint of the problem. According to Bradley et al. (1977) *shadow price* is “the change in the optimal value of the objective function per unit increase in the righthand-side value for that constraint, all other problem data remaining unchanged”. The shadow price is only positive when the corresponding constraint is binding and thus its slack or surplus is equal to zero. It is important to remember that the units of shadow prices are the units of the objective function divided by the units of the given constraint (McDill, 1999). For example, given the LP problem with an objective function that aims to maximize NPV. The shadow price of an area constraint is equal to $200. This means, that if one more unit of area were available, then the optimal value of the objective function would increase by $200. However, it is crucial to remember that if we keep adding new units of area (or any other constraint), this will not allow constant increase of the objective function value (Bettinger et al., 2009). There are certain limits, in which the shadow price value is true. At some point adding new units of area will stop improving the objective function, as it will be restricted by other input data and constraints of the model.

Reduced costs and shadow prices are important elements of the LP model sensitivity analysis. *Sensitivity analysis* refers to exploring how the optimal solution changes with the change of different model parameters (Buongiorno & Gilless, 2003). It is useful to conduct this analysis, especially given linear programming assumption of certainty (all parameters of the model are known exactly). In real-life problems there are often parameters that are just approximation and may change under different conditions. However, this may be very hard to perform sensitivity analysis by manually changing parameters and then checking the model output. Therefore, solvers run series of sensitivity analyses automatically, thus providing users with values of reduced costs and shadow prices that help to interpret the model output (IBM Corp., 2017; LINDO Systems, 2017).

In the same time, it is important to be careful about interpreting sensitivity analysis results. Sometimes slight changes in the way the constraint is written (but not changing its final meaning) can lead to completely different values of shadow prices (McDill, 2014). Moreover, reduced costs and shadow prices interpretation requires logical thinking, analysis and sometimes, additional calculations are needed (Koltai & Tatay, 2011). Especially in cases when the model has several possible optimal decision variables values combinations that provide same optimal objective function value, thus possibly also providing different sensitivity results (Koltai & Terlaky, 2000).

Practical application of shadow price values was demonstrated by Borges et al. (2014b), where shadow prices associated to area constraints were analysed. These values reflect the marginal value of each management unit for the forest owner. The younger stands had lower marginal value because they were harvested later, while the highest marginal value corresponded to the oldest stand with high productivity. This information can be very useful for the forest manager for making further decisions on how to expand the forest area. Furthermore, shadow prices demonstrate to the forest owner the maximum amount of money he or she shall consider to pay for an additional hectare of the given management unit. Similarly, it also shows the minimum price he or she shall set if selling one hectare of each management unit.

Another use of shadow prices can be the evaluation of the new products introduced to the market (Leavengood, 2000). For example, some company wished to add two mode items to their product line. They have calculated the per-unit profit of each item and estimated the amount of wood and workforce needed to produce new unit of each item. Thus, knowing shadow prices values for wood and workforce, we could estimate the opportunity cost of manufacturing one unit of each new items. If this estimated cost is lower than the calculated profit, then this item is worth producing. Mohammadi et al. (2017) used LP model and shadow prices values to decide, which species should be planted at the given study area. Chosen species demonstrated the highest shadow prices, meaning that if their plantation area is increased, they will gain higher NPV.

3. MATERIALS AND METHODS

3.1. STUDY AREA

In 2003, after the severe wildfire season in Portugal, policymakers have prescribed the creation of Areas for Forest Intervention (AFI/ZIF). They correspond to joint management areas that must encompass an area of at least 1,000 ha and 50 forest owners. The goal of AFI/ZIF creation was to promote the integration of multiple small non-industrial forest owners in order to address wildfire prevention and achieve sustainable forest management (Martins & Borges, 2007).

The study area of this work was the Zona de Intervenção Florestal (ZIF) Paiva and Entre-Douro e Sousa (ZIF_VS), which is located in the North-Western region of Portugal, approximately 50 Km from the city of Porto. The area is characterized by a Mediterranean climate with an Atlantic influence and yearly average temperatures range between 10 °C and 15 °C. The maximum elevation reaches about 700 m.

The total area of Vale do Sousa study region extends over 14,760 ha and was classified into 1,343 management units for forest management purposes (Figure 2). The landscape was classified according to the land use, the forest species composition, the stand age and the site index. The area is dominated by pure eucalypt (*Eucalyptus globulus* Labill) stands and mixed stands of eucalypt and maritime pine (*Pinus pinaster* Ait.). Hardwoods, mainly chestnut, occupy the remaining area (Table 1).

Primary ecosystem services provided by ZIF_VS are eucalypt pulpwood and maritime pine sawlogs, as well as hardwood sawlogs and carbon storage.

The study area encompasses 376 associated forest owners. About 35% of the ZIF_VS area accounts for the community (local parish) property. Over 60% of the ZIF_VS are medium and large private properties (area greater than 5 ha). Small or very small forest owners own the remaining 5%. Each landowner manages the stands with the support of the Local Forest Owners Association, which develops the joint landscape-level management plan.

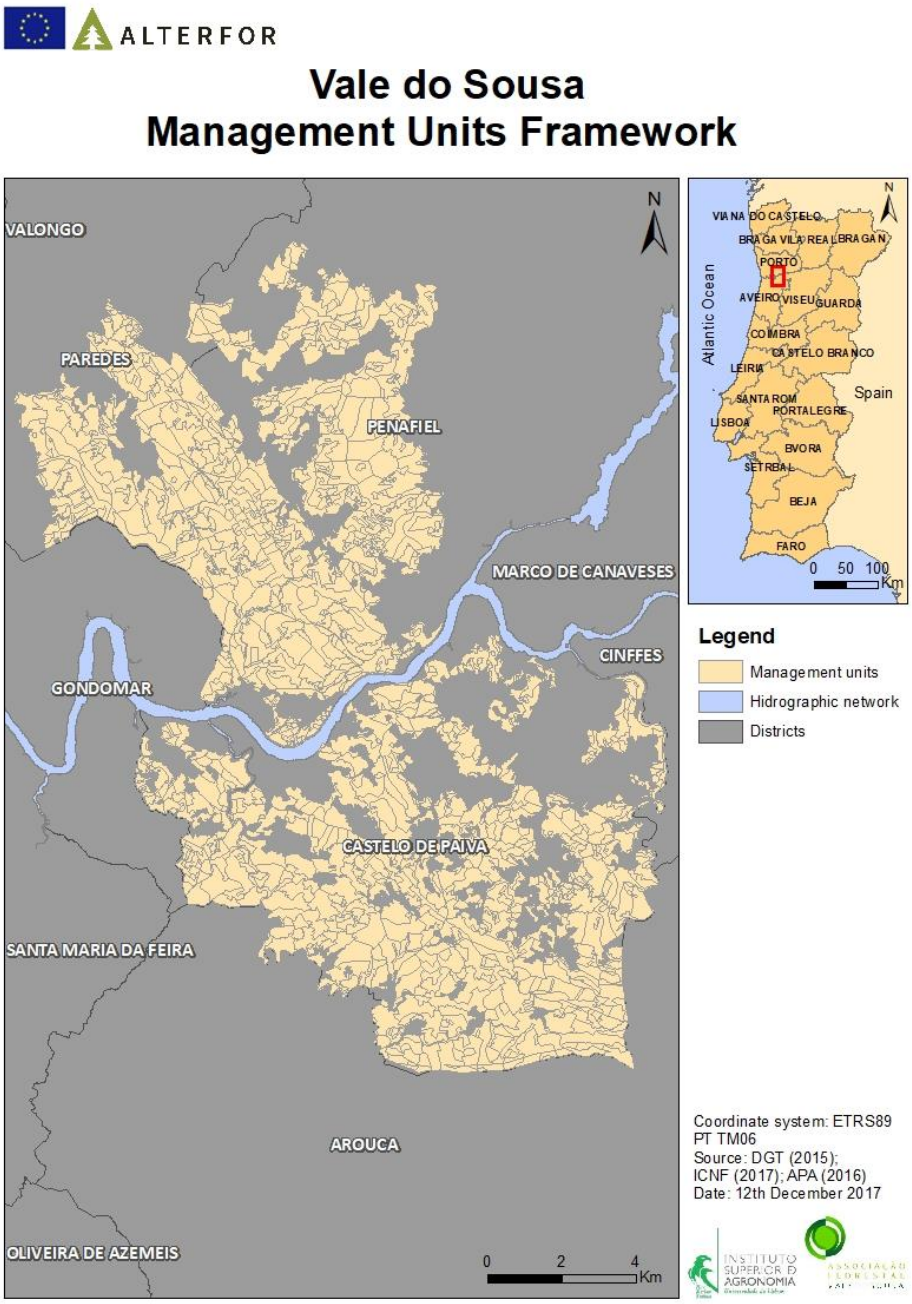


Figure 2. Location and management units framework of Zona de Intervenção Florestal (ZIF) Paiva and Entre-Douro e Sousa (ZIF_VS)

Table 1. Area distribution at the beginning of the planning horizon.

Occupation	**Area, *ha***
Mixed maritime pine and eucalypt with maritime pine dominance	318.49
Mixed maritime pine and eucalypt with eucalypt dominance	328.71
Chestnut	41.09
Eucalypt	8,645.73
Maritime pine	472.58
Riparian species	101.44
Burned	4,673.61
Shrubs	183.52
Total	14,765.17

3.2. MODEL BUILDING

A total of 250,100 stand-level prescriptions were simulated over the 90-year planning horizon with nine 10-year periods. The maritime pine prescriptions include plantations of 1111 trees per hectare with rotations of 35, 40, 45 and 50 years, and thinning occurring between age of 20 and 45 years (up to 5 years before the final cut). It also includes the alternative of resin extraction between 27 and 50 years, occurring every year until the final cut. The eucalypt prescriptions encompassed rotations based on the three coppice cycles ranging in length from 10 to 12 years and final cut by the end of the third cycle. At year 3 of second and third cycle a stool thinning option is included, which leaves in average two shoots per stool. Sweet chestnut prescriptions include rotation lengths of 40, 45, 50 and 55 years and alternative of thinning periodicities of 5 and 10 years starting from age of 15 years. Pedunculate oak prescriptions involve plantations of 1600 plants per hectare, rotations of 40, 50 and 60 years and thinning options between years 25 and 47. Cork oak prescriptions include plantations of 1600 trees per hectare, thinning at year 15, 30 and 40. First three cork removals occur at age of 30, 40 and 49 years with following removals happening every 9 years. Finally, the last prescriptions include riparian species with 1600 plants per hectare and no pruning, thinning or final cut.

Furthermore, additional prescriptions simulate the conversion of mixed maritime pine or eucalyptus stands to chestnut, pedunculate oak, cork oak or maritime pine plantations.

Additionally, all of the prescription include fuel treatment regime with periodicities of 1, 5, 10 or 15 years, as well as an option with no fuel treatment.

The stand-level prescriptions were classified into eight forest management programmes. Programme 1 – mixed maritime pine and eucalypt with maritime pine dominance; Programme 2 – mixed maritime pine and eucalypt with eucalypt dominance; Programme 3 – pure chestnut; Programme 4 – pure eucalypt stands; Programme 5 – pure maritime pine stands; Programme 6 – pure pedunculate oak plantations; Programme 7 – pure stands of cork oak; Programme 8 – riparian species.

In order to obtain the present value of ending inventory, for each combination of management unit and prescription the following steps were taken:

1. All revenues and costs occurring from the end of the planning horizon up to the end of ongoing rotation were calculated and discounted to year 90.

2. Soil Expectation Value (SEV) was obtained and discounted from the year of the end of that rotation to year 90.
3. The sum of the values computed in 1. and 2. were discounted to the beginning of the planning horizon.

In the case of cork oak, its value of ending inventory was estimated as the value of a perpetual series of cork extractions.

Thus, the value of ending inventory allows calculating the present value of all future incomes by adding it to the total net present value associated to the cash flows over the planning horizon. This new value provides the present value of the forest in case of a perpetual planning horizon.

The biodiversity indicator was computed based on the assumption that a total biodiversity score consists of two components: Score #1 "Tree species composition" that differentiate tree species with higher or lower importance for the biodiversity level and further assigns higher values for higher ages; and Score #2 "Shrub cover" that assumes the increase of biodiversity score with the increase of shrub cover (Biber et al., 2018; Botequim et al., 2014; Nieuwenhuis & Biber, 2018).

RALF – recreational and aesthetic value of the forested landscape – index indicates the level of cultural and recreational services. It was computed by ranking cultural services based on 6 key factors: stewardship, naturalness/disturbances, complexity, visual scale, historicity/imageability and ephemera (Pretzsch, 2009).

All the calculations of the coefficients for the equations of the model were done in the database system PostgreSQL (The PostgreSQL Global Development Group, 2020). In order to obtain the final equations, PHP scripts were developed (PHP Documentation Group, 2020), which connected the data from the database with decision variables.

The obtained management linear programming model is a typical Model I formulation (Johnson & Scheurman, 1977). The model can be described as follows:

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} x_{ij} = 1 \qquad i = 1 \ldots M \tag{3.1}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} pine_{ijt}\, x_{ij} = Pine\ W_t \qquad t = 1 \dots T \tag{3.2}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} eucalipt_{ijt}\, x_{ij} = Euc\ W_t \qquad t = 1 \dots T \tag{3.3}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} chestnut_{ijt}\, x_{ij} = Chest\ W_t \qquad t = 1 \dots T \tag{3.4}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} pendaok_{ijt}\, x_{ij} = POak\ W_t \qquad t = 1 \dots T \tag{3.5}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} coak_{ijt}\, x_{ij} = COak\ W_t \qquad t = 1 \dots T \tag{3.6}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} oak_{ijt}\, x_{ij} = Oak_t \qquad t = 1 \dots T \tag{3.7}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} vei_{ijt}\, x_{ij} = VEI_t \qquad t = 9 \tag{3.8}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} carb_{ijt}\, x_{ij} = Carb_t \qquad t = 1 \dots T \tag{3.9}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} npv_{ijt}\, x_{ij} = NPV_t \qquad t = 1 \dots T \tag{3.10}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} costs_{ijt}\, x_{ij} = Costs_t \qquad t = 1 \dots T \tag{3.11}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} \frac{a_i biod_{ijt} x_{ij}}{FA} = Biod_t \qquad t = 1 \dots T \tag{3.12}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} \frac{a_i ralfind_{ijt} x_{ij}}{FA} = RALF_t \qquad t = 1 \dots T \tag{3.13}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} eiv_{ij}\, x_{ij} = EIV \tag{3.14}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{Ni} A_i\, x_{ij} = A\_FMM_f \qquad f \in F \tag{3.15}$$

$$\sum_{1}^{T} Pine\ W_t = PineSawlogs \tag{3.16}$$

$$\sum_{1}^{T} Euc\ W_t = EucPulpWood \tag{3.17}$$

$$\sum_{1}^{T} Chest\ W_t = ChestSawlogs \tag{3.18}$$

$$\sum_{1}^{T} POak\,W_t = POakSawlogs \tag{3.19}$$

$$\sum_{1}^{T} COak\,W_t = COakSawlogs \tag{3.20}$$

$$PineSawlogs + EucPulpWood + ChestSawlogs + POaksawlogs + COakSawlogs = TWood \tag{3.21}$$

$$\sum_{1}^{T} Oak_t = Oak \tag{3.22}$$

$$\sum_{1}^{T} Carb_t/_T = CARB \tag{3.23}$$

$$\sum_{1}^{T} Biod_t/_T = BIOD \tag{3.24}$$

$$\sum_{1}^{T} RALF_t/_T = CULTSERV \tag{3.25}$$

$$\sum_{1}^{T} NPV_t = TNPV \tag{3.26}$$

$$\sum_{1}^{T} Costs_t = TCosts \tag{3.27}$$

$$TNPV + EIV = PVFI \tag{3.28}$$

$$0 \leq x_{ij} \leq 1 \quad \forall\, i,j \tag{3.29}$$

Where:

M is the number of management units (1343);

Ni is the number of prescriptions for each stand *i*;

a_i is the area of stand *i*;

FA is the total forested area (14,760);

T is the number of planning periods (9);

F is the number of forest management models (8);

FMM_f is the set of prescriptions that were classified as belonging to a forest management planning programme *f;*

x_{ij} is the percentage of area of management unit *i* assigned to prescription *j;*

$pine_{ijt}$ is the pine timber harvested in period *t* that results from assigning prescription *j* to the whole area of stand *i;*

$eucalipt_{ijt}$ is the eucalypt timber harvested in period *t* that results from assigning prescription *j* to to the whole area of stand *i;*

$chestnut_{ijt}$ is the chestnut timber harvested in period *t* that results from assigning prescription *j* to the whole area of stand *i;*

$pendoak_{ijt}$ is the pedunculate oak timber harvested in period *t* that results from assigning prescription *j* to the whole area of stand *i;*

$coak_{ijt}$ is the cork oak timber harvested in period *t* that results from assigning prescription *j* to the whole area of stand *i;*

oak_{ijt} is the adult cork flow in period *t* that results from assigning prescription *j* to the whole area of stand *i;*

vei_{ijt} is the standing volume of the ending inventory in stand *i* that results when assigning prescription *j* in the period *9;*

$carb_{ijt}$ is the average yearly carbon stock in period *t* that results from assigning prescription *j* to the whole area of stand *i*;

npv_{ijt} is the net present value associated to a prescription *j* in stand *i* in period *t;*

$costs_{ijt}$ is the sum of all costs associated to a prescription *j* in stand *i* in period *t;*

$biod_{ijt}$ is the biodiversity indicator in period *t* that results from assigning prescription *j* to the whole area of stand *i*, where 0 is bare land or no biodiversity and 8 is high level of biodiversity;

$ralfind_{ijt}$ is RALF index or cultural services indicator in period *t* that results from assigning prescription *j* to the whole area of stand *i*, where 1 means low cultural interest and 5 means high cultural and recreation interest;

eiv_{ij} is the value of ending inventory associated to a prescription *j* in stand *i* in period *t;*

A_FMM_f is the area assigned to forest management model f.

Equations 3.1 state that the total percentage of area assigned to prescriptions must be equal to one. Equations 3.2, 3.3, 3.4, 3.5 and 3.6 define, respectively, the pine, eucalypt, chestnut, pedunculate oak and cork oak timber yield in each planning period. Equations 3.7 define the adult cork yield in each planning period. Equations 3.8 define the standing volume in the case study area at the end of the planning horizon. Equations 3.9 define the average carbon stock in the study area in each planning period. Equations 3.10 and 3.11 define respectively the net present value and costs in each planning period. Equations 3.12 and 3.13 define respectively biodiversity indicator and cultural services level represented by RALF-index. Equations 3.14 define the ending inventory value. Equations 3.15 define the area assigned to each forest management program.

Equations 3.16 to 3.25 represent respectively the total pine saw logs yield, the total eucalypt pulpwood yield, the total chestnut saw logs yield, the total pedunculated oak saw logs yield, the total cork oak saw logs yield, the total wood yield, the total adult cork yield, the average carbon stock, the average biodiversity indicator and the average cultural services indicator. Equations 3.26 to 3.28 define respectively the total net present value, total costs and the net present value of all future incomes. The inequalities 3.29 are the non-negativity constraints.

For solving the linear programming model, CPLEX software was used (IBM Corp., 2017).

3.3. SCENARIOS

A total number of 16 scenarios based on the described model were considered for the analysis.

Scenario 1-4:

Considers the basic LP model described in 3.2. without constraints about forest management. Scenario 1 corresponds to the objective function to maximize net present value (NPV), Scenario 2 – to minimize costs, Scenario 3 – to maximize ending inventory value (EIV) and Scenario 4 – to maximize present value of all future incomes (PVFI).

Scenario 5-8:

Considers the basic model formulation with addition of even-flow constraints concerning the timber harvested in each period of the planning horizon. The constraints ensure that the fluctuation of the harvested timber does not exceed 10%.

In order to formulate even-flow constraints additional accounting variables *TimbHarv*$_t$, which represent the total timber harvested in each period, were introduced:

$$PineW_t + EucW_t + ChestW_t + POakW_t + COakW_t = TimbHarv_t \qquad t = 1 \ldots T(9) \tag{3.30}$$

Thus, the final even-flow constraints are:

$$TimbHarv_{t+1} - 1.1\, TimbHarv_t \leq 0 \tag{3.31}$$

$$TimbHarv_{t+1} - 0.9\, TimbHarv_t \geq 0 \tag{3.32}$$

Each of the four scenarios corresponds to respectively objective function of maximization of NPV, minimization of costs, maximization of EIV and maximization of PVFI.

Scenario 9-12:

Considers the basic model formulation with addition of target constraints. The targets were set for the total harvested timber in each period. For the formulation of the constraints additional accounting variables defined in equations 3.30 were considered. The constraints ensure the wood production to be at least 1,000,000 m^3 in each period of the planning horizon:

$$TimbHarv_t \geq 1\,000\,000 \tag{3.33}$$

Each of the four scenarios corresponds to respectively objective function of maximization of NPV, minimization of costs, maximization of EIV and maximization of PVFI.

<u>Scenario 13-16:</u>

Considers the basic model formulation with addition of target constraints of total timber yield of each tree species. The constraints assure that total yield of maritime pine wood is at least 3,000,000 m^3, eucalypt pulpwood – 6,000,000 m^3, chestnut wood - 200,000 m^3, pedunculate oak - 10,500 m^3 and cork oak - 250,000 m^3. Thus, the equations are:

$$PineSawlogs \geq 3\,000\,000 \tag{3.34}$$

$$EucPulpWood \geq 6\,000\,000 \tag{3.35}$$

$$ChestSawlogs \geq 200\,000 \tag{3.36}$$

$$POakSawlogs \geq 10\,500 \tag{3.37}$$

$$COakSawlogs \geq 250\,000 \tag{3.38}$$

Each of the four scenarios corresponds to respectively objective function of maximization of NPV, minimization of costs, maximization of EIV and maximization of PVFI.

4. RESULTS

Results of 16 linear programming models demonstrated that the highest value of the objective function of maximizing NPV (62.82×10^{6} €), minimizing costs (11.84×10^{6} €), maximizing EIV (7.88×10^{6} €) and maximizing PVFI (68.74×10^{6} €) were obtained in Scenarios 1, 2, 3 and 4 respectively, cases when no additional constraints are applied (Table 1). The second best optimal values of the objective function in case of NPV, EIV and PVFI maximization scenarios, were observed in Scenario 13 (62.14×10^{6} €), Scenario 15 (6.95×10^{6} €) and Scenario 16 (67.91×10^{6} €) respectively, where timber target constraints per species are applied. The reason must be the fact that these constraints are only setting the minimum amount of timber to be harvested over the whole planning horizon, thus allowing the model to allocate operations freely over the periods. In case of cost minimization, the second best result was observed for Scenario 6 (12.94×10^{6} €), where 10% timber even-flow constraints are included, as they did not set the minimum timber yield. Moreover, the model had more freedom in choosing the species, this way a significant decrease of cork oak plantations was observed (Figure 3).

For all of the models, the major part of the area was prescribed to forest management Programme 5 and 4, which encompass pure stands of maritime pine and eucalypt respectively. This result was expected, as incomes that result from the sale of both maritime pine and eucalypt timber often exceed management costs. The next more common prescriptions are pure stands of chestnut (Programme 3) and cork oak (Programme 7) (Figure 3 and 4). However, in Scenario 3 most of the area was assigned to Programme 5, followed by Programme 3 and then some hectares to Programme 4. The highest number of hectares assigned to chestnut was observed in Scenario 7. Cork oak prescriptions were chosen for all models, with highest number of hectares demonstrated in Scenario 5 and lowest (less than one hectare) – in Scenario 2. All models included some hectares assigned to Programmes 1 and 2 (mixed stands of maritime pine and eucalypt with respectively pine and eucalypt dominance). However, in both mixed programmes eucalypt was replaced by other species. Pedunculate oak prescriptions (Programme 6) were implemented only in Scenario 7 and when target constraints for timber per species were included in Scenarios 13-16. Riparian species (Programme 8) are included in all scenarios with minimum hectares to be assigned according to area constraints of the models.

Table 2. Summary of solution of 16 LP models.

Criteria	Scenario							
	1	2	3	4	5	6	7	8
NPV (10^6 €)	62.82	46.22	40.08	62.47	59.49	45.28	43.71	59.29
Costs (10^6 €)	33.10	11.84	31.85	33.41	37.14	12.94	42.32	37.42
EIV (10^6 €)	5.53	4.15	7.88	6.26	4.14	3.08	5.66	4.49
Pine	3.79	3.17	6.98	4.18	2.53	2.00	2.72	2.75
Eucalypt	1.61	0.98	0.15	1.84	1.14	1.03	1.61	1.21
Chestnut	0.002	0.002	0.48	0.004	0.22	0.002	1.01	0.23
Peduncu-late oak	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Cork oak	0.13	0.001	0.28	0.23	0.24	0.04	0.31	0.31
PVFI (10^6 €)	68.36	50.37	47.96	68.74	63.62	48.36	49.37	63.79
Timber (10^6 m^3)	10.29	9.81	6.70	10.15	10.29	10.31	9.36	10.27
Carbon (10^8 Kg)	19.98	3.71	3.52	19.93	29.44	7.51	7.80	26.53
Cork (15×10^6 Kg)	1.09	0.0003	0.21	1.07	1.77	0.25	0.36	1.61
Biodiversity index	2.69	2.48	2.98	2.69	2.79	2.52	2.86	2.77
RALF-index	3.03	3.02	3.01	3.03	3.04	3.02	3.08	3.04

Table 2. Summary of solution of 16 LP models (continuation)**.**

Criteria	Scenario							
	9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
NPV (10^6 €)	59.39	50.16	53.68	59.15	62.14	48.22	53.93	61.82
Costs (10^6 €)	36.79	14.29	41.44	36.64	35.57	13.50	37.55	35.62
EIV (10^6 €)	4.06	3.74	5.13	4.97	5.38	3.87	6.95	6.10
Pine	2.72	2.32	3.31	3.45	3.62	2.24	4.23	3.97
Eucalypt	1.02	1.02	1.02	1.02	1.61	1.37	2.04	1.84
Chestnut	0.14	0.24	0.52	0.22	0.04	0.04	0.29	0.10
Pedunculate oak	0.00	0.00	0.00	0.00	-0.01	0.004	0.01	-0.01
Cork oak	0.19	0.16	0.29	0.27	0.13	0.22	0.39	0.19
PVFI (10^6 €)	63.45	53.91	58.82	64.12	67.52	52.09	60.88	67.91
Timber (10^6 m^3)	10.53	10.21	10.24	10.37	10.28	10.14	9.56	10.14
Carbon (10^8 Kg)	26.81	25.53	20.68	24.80	20.09	20.88	13.18	20.06
Cork (15×10^6 Kg)	1.48	1.35	1.18	1.40	1.09	0.90	0.99	1.07
Biodiversity index	2.74	2.71	2.70	2.72	2.73	2.66	2.73	2.72
RALF-index	3.02	3.03	3.03	3.03	3.03	3.03	3.03	3.03

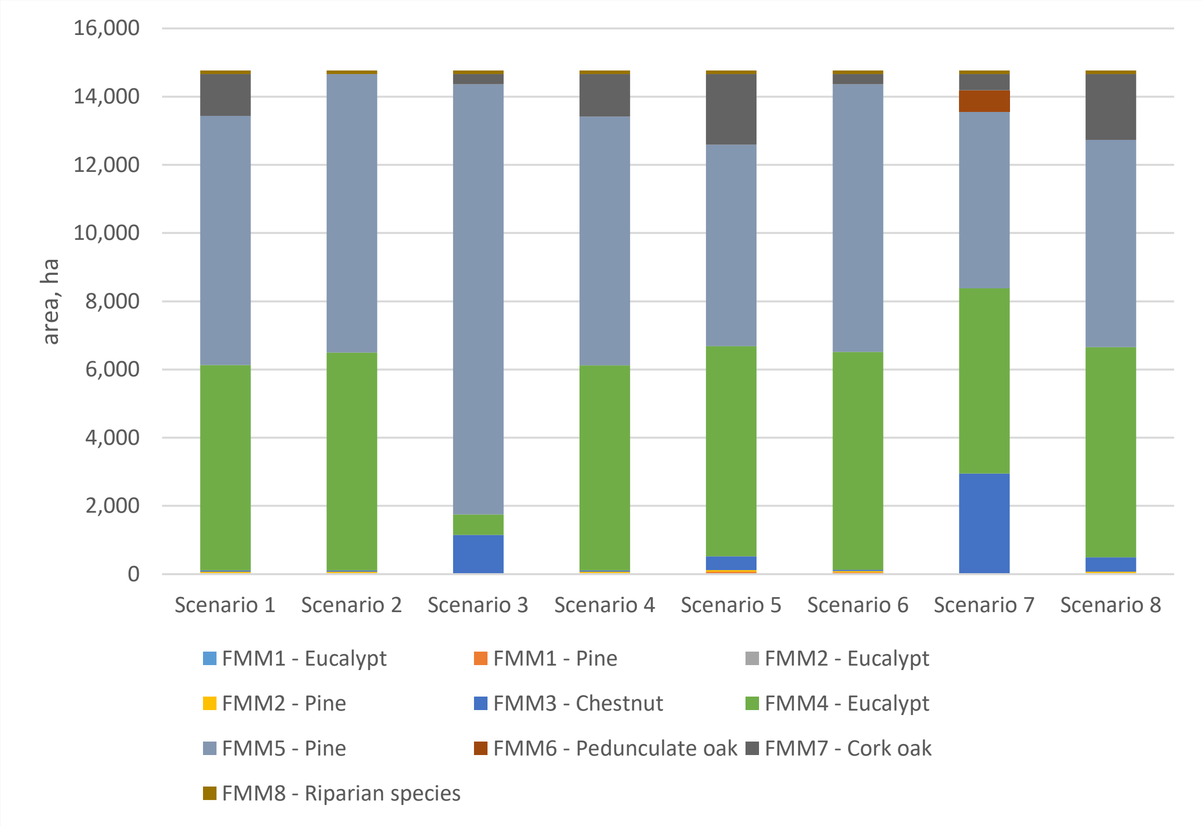


Figure 3. Area distribution per forest management programme for Scenario 1-8.

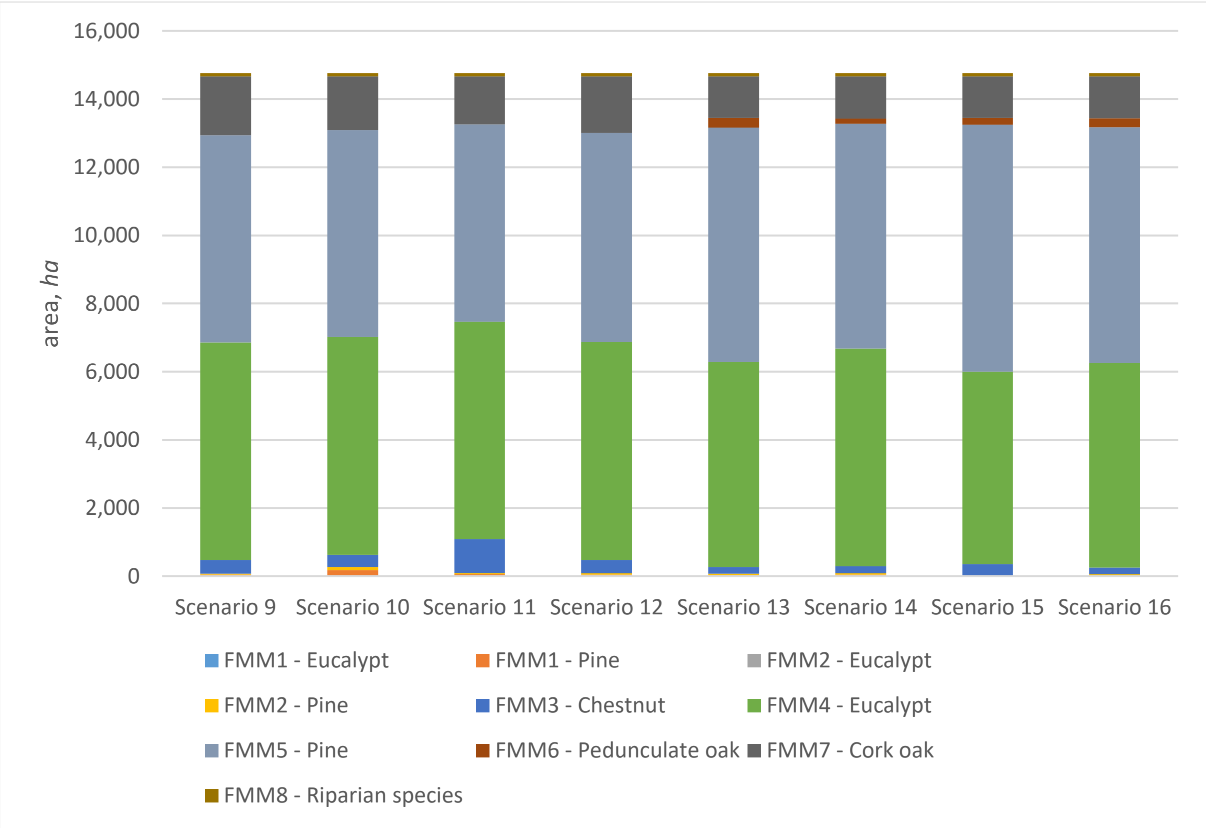


Figure 4. Area distribution per forest management programme for Scenario 9-16.

In comparison to the initial inventory at the beginning of planning horizon, all burned lands and areas covered with shrubs were converted to other species (Table 3). In all cases the area assigned to Programme 1, 2 and 4 decreases. In case of Programme 3, the number of hectares is either the same, or increases, while a significant increase of area assigned to Programme 5 is observed in all Scenarios. Pedunculate oak (Programme 6) and Cork oak (Programme 7) were the new management programmes introduced. In all cases riparian species (Programme 8) occupy the same area as in the beginning of the planning horizon.

Table 3. Area assigned to each Forest Management Programme (ha) in comparison to initial distribution.

Scenario	Forest Management Programme								Burned or shrubs
	1	2	3	4	5	6	7	8	
Initial	318.49	328.71	41.09	8,645.73	472.58	0	0	101.44	4,857.13
1	35.69	34.91	41.09	6,014.95	7,311.71	0	1,225.30	101.44	0
2	35.69	34.91	41.09	6,386.87	8,164.79	0	0.31	101.44	0
3	7.24	7.42	1,132.16	603.59	12,617.30	0	295.94	101.44	0
4	35.69	34.72	41.09	6,008.40	7,293.30	0	1,250.45	101.44	0
5	76.57	42.26	403.37	6,159.31	5,912.54	0	2,069.60	101.44	0
6	56.01	34.91	41.09	6,386.87	7,842.63	0	302.16	101.44	0
7	7.24	9.36	2,934.91	5,429.79	5,171.68	627.98	482.72	101.44	0
8	35.69	37.46	421.53	6,159.19	6,075.64	0	1,934.14	101.44	0
9	39.83	34.91	398.56	6,386.87	6,082.19	0	1,721.29	101.44	0
10	179.63	92.27	354.31	6,386.87	6,073.65	0	1,576.92	101.44	0
11	51.95	42.14	990.81	6,386.24	5,788.20	0	1,404.33	101.44	0
12	44.72	34.91	398.56	6,386.87	6,134.88	0	1,663.71	101.44	0
13	35.69	34.91	195.68	6,014.95	6,886.84	281.18	1,214.42	101.44	0
14	42.93	42.32	204.59	6,386.87	6,598.12	154.24	1,234.58	101.44	0
15	7.24	7.42	339.24	5,649.18	7,242.25	200.68	1,217.63	101.44	0
16	20.83	30.09	199.26	6,008.40	6,911.79	270.14	1,223.16	101.44	0

According to the total EIVs of all the models, most of the ending inventory is concentrated in areas with maritime pine stands, followed by eucalypt. Cork oak and chestnut stands bring much lower contribution. Pedunculate oak, even when its prescriptions are included, contributes very little (Scenario 7, 14 and 15), or even negatively (Scenario 13 and 16). This is due to the distribution of area per forest species at the end of the planning horizon and low (often negative) values of ending inventory of pedunculate oak stands.

Shadow prices corresponding to area constraints were analysed. They range differently for each scenario. For scenarios with no constraints, ranges are between -1,357.44 and 15.415.82 for Scenario 1; 0 and 1,923.28 for Scenario 2; between 0 and 14,571.38 for Scenario 3; -1,342.62 and 16,768.37 for Scenario 4. For scenarios with 10 % timber even-flow constraints shadow prices ranges are between -3,406.23 and 38,522.02 for Scenario 5; -7,182.81 and 3,150.77 for Scenario 6; -182.43 and 14,363.01 for Scenario 7; -3,487.02 and 41,584.41 for Scenario 8. For scenarios with timber target per period, shadow prices range between -1,323.33 and 52 799.79 for Scenario 9; -42,697.98 and 1,910.80 for Scenario 10; 0 and 17,202.27 for Scenario 11; -1 299.17 and 66,730.60 for Scenario 12. Finally, for scenarios with timber target constraints for species the ranges lie between -1,357.44 and 16,092.49 for Scenario 13; -1,953.93 and 1,844.79 for Scenario 14; 0 and 14,758.53 for Scenario 15; -1,342.62 and 16,768.37 for Scenario 16. Units of shadow prices that correspond to area constraints are euros per hectare. The stands with highest shadow prices values are all characterized by high site index. In Scenarios 1, 2, 5, 6 and 8 highest value stands are with maritime pine or conversion to maritime pine. In Scenarios 3, 4, 7, 11, 15 and 16 they are conversions to cork oak stands. In Scenarios 9, 10 and 12 they are pure eucalypt stands. Finally, in Scenario 13, it is a stand with conversion to pedunculate oak and in Scenario 14, it is a pure stand of chestnut. Therefore, it is possible to conclude that maritime pine and eucalypt stands have higher value mostly in cases, when the objective function was to maximize NPV, minimize costs or maximize PVFI, as these stands are normally characterized by higher revenues. Stands with cork oak are highly valuable when the objective function is to maximize EIV or PVFI due to high values of its ending inventory. Finally, pedunculate oak and chestnut stands become most valuable when timber target constraints per species are introduced.

The total volume of harvested timber tend to vary according to the objective function in Scenarios 1-4, when no management constraints are applied. However, when certain timber related constraints are included in all the other scenarios, it leads to relatively similar total timber yield. Among all models, the highest amount of harvested timber was observed in Scenario 9 (10.53×10^6 m^3); however, this model does not demonstrate the highest NPV. Lower timber yield is normally observed for scenarios with the objective function of maximizing EIV, with the lowest volume of harvested timber in Scenario 3 (6.70×10^6 m^3).

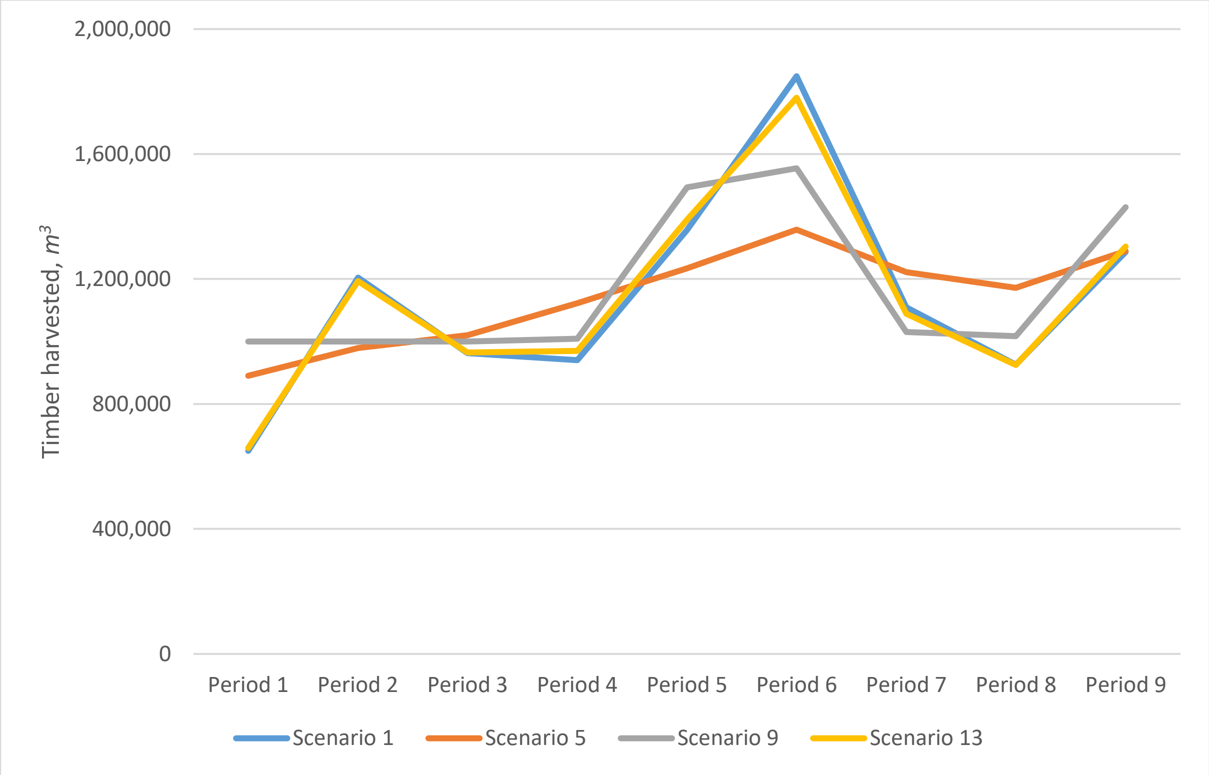


Figure 5. Timber flow for Scenario 1, 5, 9 and 13.

Addition of management related constraints change the flow of timber over the periods. For example, let us consider scenarios with the objective of maximizing NPV. In Scenario 1 we can observe that timber is not equally distributed over the periods, with extreme peaks in the Period 6, then Period 9 and 2 (Figure 5). When we introduce 10% even-flow constraints in Scenario 5, we observe that timber yield gradually increases until Period 6, then slightly decreases and grows a little again in Period 9. Nevertheless, the total wood volume for these two scenarios is the same (10.29×10^6 m^3). Further, when we introduce target constraints for each period in Scenario 9, we can observe that in Periods 1, 2 and 3 it is harvesting exactly the minimum required by the model, with slight increase in Period 4. Then much higher yield observed in the next two periods with the following decrease to almost the minimum required, until further increase in Period 9. However, when we include timber target constraints per species in Scenario 13, the timber flow over the periods follows the pattern more similar to Scenario 1, with peaks in Period 6, 9 and 2. To conclude, in all cases there is generally some increase of timber in Period 2 due to the age distribution of the existing stands that were harvested in this period. Next, there is always a substantial increase in Period 5 and highest peak in Period 6 due to the accumulation of wood from new stands that were planted during first periods, followed by a significant decrease until Period 9, when the next big accumulation of wood happens.

For Scenarios 5-8 even-flow constraints were analysed to discover if they are bounding and what are the shadow prices. In both Scenario 5 and Scenario 8, bounding constraints were the upper bounds (timber increase up to 10% from timber harvested in the previous period) for pairs of periods 1 and 2, 3 and 4, 4 and 5, 5 and 6, 8 and 9. In the same time, lower bound (timber decrease up to 10% from timber harvested in the previous period) was met for the pair of 6 and 7 periods. The highest shadow prices values in both Scenarios 5 and 8 were observed for upper bound constraint for pair of 3 and 4 period (55.05 € and 60.45 € respectively). The lowest shadow price for upper bound of pair of 8 and 9 periods (1.04 € and 1.11 € respectively for Scenario 5 and 8). In Scenario 6, bounding constraints were only upper bounds for pair of periods 1 and 2, 3 and 4, 4 and 5, 5 and 6. Shadow prices vary between -18.84 € (periods 3 and 4) and -2.81 € (periods 1 and 2). In Scenario 7, bounding constraints were upper bounds for periods 1 and 2, 3 and 4, 5 and 6; lower bounds for periods 6 and 7, 7 and 8. Shadow prices are relatively low and range between 0.50 € (periods 7 and 8) and 6.49 € (periods 3 and 4). From this information, it is possible to see the correlation with the timber flows discussed above. In general, bounding upper bound constraints suggest us that it is possible to harvest more in certain periods and this way improve the objective function. Namely, those periods are Periods 2, 4, 5, 6, 9 for Scenarios 5 and 8; Periods 2, 4, 5 and 6 for Scenario 6; Periods 2, 4 and 6 for Scenario 7. All of these periods correspond to a significant increase of harvested timber, while bounding lower bound constraints suggest harvesting less in Period 7 and 8 after the highest peak in Period 6. Highest shadow prices in all cases correspond to the opportunity to harvest more in Period 4 due to the fact that wood is already accumulating at this period, followed by lower, but still significant shadow prices of harvesting more in Period 6, where the highest peak is observed.

Timber target constraints per period in Scenario 9-12 and shadow prices that correspond to them were analysed. The similar pattern was observed in Scenarios 9 and 12, where bounding constraints were observed only in Periods 1, 2 and 3. The corresponding shadow prices in Scenario 9 are -76.09 €/m^3, -59.38 €/m^3 and -46.30 €/m^3. In Scenario 12, shadow prices for constraints of firth three periods are -102.06 €/m^3, -79.62 €/m^3 and -63.63 €/m^3. Constraints for all the other periods are non-binding, with the lowest slack in Period 4 and highest slack in Period 6. In Scenario 10 binding constraints correspond to Periods 1, 2, 3, 4 and 8 with shadow prices equal to 79.79 €/m^3, 63.04 €/m^3, 52.86 €/m^3, 0.50 €/m^3 and 0.36 €/m^3 respectively. The lowest slack is in Period 7 and highest slack in Period 6. Finally, binding target constraints in

Scenario 11 are for Periods 1, 2, 3 and 8 with shadow price values of -27.79 €/m^3, -22.16 €/m^3, -18.45 €/m^3 and -0.51 €/m^3 respectively. The lowest slack is observed in Period 4 and the highest – in Period 6. Thus, we can see that in all cases harvesting more in earlier periods, namely Period 1, will lead to the negative influence on the objective function optimal value, because young stands are prevalent in that period. The highest slack is always observed in Period 6, when the highest wood volume accumulation is taking place.

Results of the analysis of timber target constraints per species in Scenarios 13-16 were further presented. In Scenarios 13 and 16 bounding constraints were the ones that correspond to pedunculate oak and chestnut yield. The shadow prices are -93.71 €/m^3 and -99.80 €/m^3 for pedunculate oak and -0.18 €/m^3 and -0.45 €/m^3 for chestnut in Scenario 13 and 16 respectively. In both scenarios, the highest slack is observed for eucalypt while the lowest slack is associated to cork oak. In Scenario 14 bounding constraints correspond to maritime pine, pedunculate oak, chestnut and cork oak. The corresponding shadow prices are 0.37 €/m^3, 25.19 €/m^3, 1.89 €/m^3 and 5.99 €/m^3. Finally, bounding constraints in Scenario 15 are for eucalypt, pedunculate oak, and cork oak, shadow prices are -0.26 €/m^3, -7.99 €/m^3 and -1.33 €/m^3 respectively. Maritime pine has higher slack, than chestnut. It is possible to conclude, that harvesting additional volume of pedunculate oak will always lead to highly negative influence on the objective function value and much lower effect from additional volume of cork oak and chestnut.

The total carbon stock varies substantially among all the Scenarios, with the highest value in Scenario 5 (29.44×10^8 Kg) and lowest value in Scenario 2 (3.71×10^8 Kg). In Scenarios 1-8 the carbon stock tends to be much lower, when the objective is to minimize costs or to maximize EIV. However, this is not the case for Scenarios 9-16. Thus, in the case of models that maximize NPV and maximize PVFI, the highest values of carbon stock are observed when 10% timber even-flow constraints are applied (Scenarios 5 and 8 respectively). On the other hand, in the case of models that minimize costs and maximize EIV, the highest carbon stock is obtained when timber target constraints per period are applied (Scenarios 9 and 10 respectively).

The amount of cork extracted reflects the number of hectares of cork oak assigned to each scenario. For instance, the lowest amount of cork is in Scenario 2 ($0.3\times15\times10^3$ Kg), where less than one hectare of cork oak is chosen. Accordingly, the highest amount of cork ($1.77\times15\times10^6$ Kg) is observed in Scenario 5, where the highest number of hectares for cork oak were assigned.

The analysis considered too the biodiversity indicator and the cultural services RALF-indicator. The former varies over scenarios from 2.48 up to 2.86 on the scale from 0 to 8, suggesting low level of biodiversity in all cases. The RALF-index varies from 3.01 to 3.08 on the scale from 0 to 5, suggesting moderate cultural and recreational interest.

5. DISCUSSION

To achieve the goal of the thesis, 16 linear programming models (Scenarios) were formulated. Scenarios were obtained in two ways: by changing the objective function of the model and by the addition of management related constraints. Four objective functions were tested: maximizing the net present value (NPV), minimizing costs, maximizing ending inventory value (EIV) and maximizing the present value of all future incomes (PVFI), which consists of NPV and EIV. Four variations of constraints were applied: 10% even-flow of timber (Scenarios 5-8), targets for timber harvested per period (Scenarios 9-12), targets for timber harvested per species (Scenarios 13-16). Scenarios 1-4 did not include policy constraints.

All models were solved with the CPLEX software, which proved to be an excellent tool for processing large long-term planning linear programming problems with thousands decision variables. From the output files the values of the objective functions were extracted, together with timber volumes, number of hectares assigned to each management programme, total carbon stock, total amount of cork extracted, biodiversity index and, finally, recreational and cultural interest index. Furthermore, shadow prices that correspond to area constraints, 10% even-flow constraints and timber target constraints per period and per species were also considered for the analysis and important information has been obtained from them. Each model was solved and its output analysed separately with further comparison to other scenarios to gain insights about the management problem.

Analysis of models with the same constraints, but different objective functions, allowed us to see the trade-offs between the four economic criteria we have studied. In all of the cases, the change of the objective function from maximizing NPV to minimizing costs, leads to significant decrease of not only costs, but also NPV (up to 26%), EIV (up to 24%) and PVFI (up to 26%). While maximizing EIV, in comparison to maximizing NPV, brings slightly higher costs (up to 3%), but significantly lower NPV (up to 36%) and PVFI (up to 29%). Similar results for trade-offs between NPV and EIV were obtained by Borges et al. (2014a). Scenarios with objective function of maximizing PVFI demonstrated to be very similar to scenarios with maximizing NPV due to the fact that when maximizing PVFI the model actually simultaneously maximizes both NPV and EIV. Furthermore, notably lower, but still substantial difference between economic criteria depending on the objective function, were observed in Scenarios 9-12, where timber target constraints per period are applied.

Introduction of different management related constraints is important for reaching sustainable forest management. For instance, adding 10% timber even-flow constraints leads to the increase of total carbon stock, same was proved by Keleş et al. (2007). In the same time, increasing the amount of harvested timber and extracted cork. However, the economic criteria are getting lower (higher in case of costs), but not dramatically. Furthermore, introduction of both timber target constraints per period and per species also leads to increased carbon stock, much higher in case of maximization of EIV and minimization of costs, than even-flow constraint.

The results of area distribution and analysis of shadow prices that correspond to area constraints and timber target constraints per species demonstrated that the most valuable stands are with maritime pine and eucalypt, followed by cork oak and chestnut. Pedunculate oak prescriptions are very unlikely to be chosen, unless related constraints are applied, such as in Scenarios 13-16. Similarly, much lower areas for cork oak stands are prescribed, when the objective function is to minimize costs and maximize EIV and there are no constraints that force the model to choose them.

The analysis of timber flows and shadow prices that correspond to even-flow constraints and timber target per period showed that most of timber is accumulated in four periods. Firstly, in Period 2, from current inventory, then in Periods 5 and 6, from new plantations, with the highest peak always observed in Period 6. Finally, there is always a significant shortage of timber harvested in the following two periods, resulting to a new peak in Period 9. The difference between amounts of timber per period is more significant when no constraints are applied or timber targets per species are introduced. Timber targets per period and even-flow constraints can guarantee more stable wood yield over the planning horizon.

The level of biodiversity, as well as the level of cultural and recreational services, do not change substantially across scenarios, remaining medium or even low. This suggests us that, if corresponding constraints are not applied, the level of these ecosystem services will remain the same for all of the cases, especially when the objective function is connected to economic criteria. Holland et al. (1994) have demonstrated that when the LP model objective function is to maximize NPV, it always leads to substantial decrease of the diversity level. Similarly, Zhou & Gong (2004) proved that including environmental related constraints will lead to a decrease of NPV.

6. CONCLUSIONS

Forest management is an extremely complex process, during which environmental, economic and social aspects must be considered. Linear programming is a powerful technique that can assist decision makers in these issues. In this work, the application of linear programming allowed studying the study forest area of Vale do Sousa, its economic criteria values, wood flows and periods of wood accumulation. It provided further information about the most valuable stands and species, as well as about the level of other non-wood ecosystem services and how they are dependent on other elements of the model. The objective of the thesis was achieved by application of changes in LP model formulation and by the analysis of the corresponding solutions.

This study showed that the objective function for forestry planning problems should be chosen carefully. For example, if it would be to minimize costs or maximize ending inventory value (EIV), it is crucial to understand, that other criteria may significantly decrease as well, especially the net present value (NPV), thus the profit from the forest. Similarly, if we want to get the maximum possible profit, we have to understand that we may have to invest more. Objective function to maximize present value of all future incomes (PVFI) is a good compromise between maximizing NPV and maximizing EIV.

Furthermore, certain management constraints need to be applied. Firstly, tree species distribution should be considered, depending if the forest owner is only interested to have the most profitable species, or to have some diversity. Here we can apply timber targets per species or area constraints that correspond to each management programme. Secondly, timber even-flow or timber target per period might be of big interest for forest owners, as this way the stable profits can be guaranteed for them. Human factor shall be taken into account, as people are normally interested in receiving money as soon as possible, not waiting until year 60, when the most valuable wood is accumulated, as in our case study. Finally, to address the level of other ecosystem services, apart from wood, such as biodiversity and cultural services, it is also recommended to set certain constraints, otherwise they might stay at lower levels. Nevertheless, carbon stock depends significantly on the objective function and management related constraints.

However, applying many constraints related to each ecosystem service may require the setting of targets by the decision makers before they have access to information about trade-offs between those services. Thus, another way to address these issues is to combine linear programming technique with multiple criteria decision methods (Diaz-Balteiro & Romero, 2008), such as Pareto frontier (Borges et al., 2014a; Lotov et al., 2004; Marques et al., 2017) or Data Envelopment Analysis (Joro et al., 1998).

Linear programming is a non-spatial technique. Thus, if model assigns 50% of area of the stand to one prescription and other 50% to another, we do not have information about how exactly the area should be divided. Therefore, another possible issue to address for further studies is overcoming spatial limitation of linear programming technique (Gustafson et al., 2006; Öhman & Eriksson, 2002).

7. BIBLIOGRAPHY

Bettinger, P., Boston, K., Siry, J. P., & Grebne, D. L. (2009). *Forest managemnet and planning*. Academic Press.

Biber P, Maarten TN, Black K, Borga M, Borges J G, Felton A ,Hoogstra-Klein M, Lindbladh M, Zoccatelli D. (2018). ALTERFOR WP3, Deliverable 3.2. - Synthesis report: Discrepancies between ES needs and ES outputs under current FMMs, 44p. Retrieved December 7, 2020 from https://alterfor-project.eu/files/alterfor/download/Deliverables/D3.2%20Synthesis%20report.pdf

Borges, J. G., Garcia-Gonzalo, J., Bushenkov, V., McDill, M. E., Marques, S., & Oliveira, M. M. (2014a). Addressing multicriteria forest management with Pareto frontier methods: An application in Portugal. *Forest Science*, *60*(1), 63–72. https://doi.org/10.5849/forsci.12-100

Borges, J. G., Garcia-Gonzalo, J., Marques, S., Valdebenito, V. A., McDill, M. E., & Falcão, A. O. (2014b). Strategic Management Scheduling. In J. G. Borges, L. Diaz-Balteiro, M. E. McDill, & L. C. E. Rodriguez (Eds.), *The Management of Industrial Forest Plantations: Theoretical Foundations and Applications*. Elsevier. https://doi.org/10.1007/978-94-017-8899-1_5

Borges, J. G., Marques, S., Garcia-Gonzalo, J., Rahman, A. U., Bushenkov, V., Sottomayor, M., Carvalho, P. O., & Nordström, E.-M. (2017). A Multiple Criteria Approach for Negotiating Ecosystem Services Supply Targets and Forest Owners' Programs. *Forest Science*, *63*(1), 49–61. https://doi.org/10.5849/fs-2016-035

Borges, J. G., Oliveira, Â. C., & Costa, M. A. (1997). A quantitative approach to cork oak forest management. *Forest Ecology and Management*, *97*(3), 223–229. https://doi.org/10.1016/S0378-1127(97)00064-9

Botequim, B., Zubizarreta-Gerendiain, A., Garcia-Gonzalo, J., Silva, A., Marques, S., Fernandes, P. M., Pereira, J. M. C., & Tomé, M. (2014). A model of shrub biomass accumulation as a tool to support management of portuguese forests. *IForest*, *8*, 114–125. https://doi.org/10.3832/ifor0931-008

Bradley, S. P., Hax, A. C., & Magnanti, T. L. (1977). *Applied Mathematical Programming*. Addison-Wesley.

Buongiorno, J., & Gilless, J. K. (2003). *Decision Methods for Forest Resource Management*. Academic Press.

Burstein, F., & Holsapple, C. (2008). *Handbook on Decision Support Systems 1: Basic Themes*. Springer-Verlag London Ltd.

Campanella, S., Corsano, G., & Montagna, J. M. (2018). A modeling framework for the optimal forest supply chain design considering residues reuse. *Sustainable Production and Consumption*, *16*, 13–24. https://doi.org/https://doi.org/10.1016/j.spc.2018.06.003

Dantzig, G. B., & Thapa, M. N. (2006). *Linear programming 1: introduction.* Springer Science & Business Media.

Davis, L. S., Johnson, K. N., Bettinger, P., & Howard, T. (2001). *Forest Management*. McGraw Hill Publishing Company.

Demirci, M., & Bettinger, P. (2015). Using mixed integer multi-objective goal programming for stand tending block designation: A case study from Turkey. *Forest Policy and Economics*, *55*, 28–36. https://doi.org/https://doi.org/10.1016/j.forpol.2015.03.007

Diaz-Balteiro, L., & Romero, C. (2008). Making forestry decisions with multiple criteria: A review and an assessment. *Forest Ecology and Management*, *255*(8–9), 3222–3241. https://doi.org/10.1016/j.foreco.2008.01.038

Eriksson, L. O., & Borges, J. G. (2014). Computerized decision support tools to address forest management planning problems: history and approach for assessing the state of art world-wide. In L. O. Eriksson, J. Garcia-Gonzalo, A. Trasobares, T. Hujala, E. M. Nordström, & J. G. Borges (Eds.), *Computer-Based Tools for Supporting Forest Management: The Experience and the Expertise World-wide* (pp. 3–25).

Fuentealba, S., Pradenas, L., Linfati, R., & Ferland, J. A. (2019). Forest harvest and sawmills: An integrated tactical planning model. *Computers and Electronics in Agriculture*, *156*, 275–281. https://doi.org/https://doi.org/10.1016/j.compag.2018.11.011

Garcia, O. (1990). Linear programming and related approaches in forest planning. *New Zealand Journal of Forestry Science*, *20*(3), 307–331.

Gustafson, E. J., Roberts, L. J., & Leefers, L. A. (2006). Linking linear programming and spatial simulation models to predict landscape effects of forest management alternatives. *Journal of Environmental Management*, *81*(4), 339–350. https://doi.org/https://doi.org/10.1016/j.jenvman.2005.11.009

Hillier, F. S., & Lieberman, G. J. (2010). *Introduction to operations research* (9th editio). McGraw-Hill.

Holland, D. N., Lilieholm, R. J., Roberts, D. W., & Gilless, J. K. (1994). Economic trade-offs of managing forests for timber production and vegetative diversity. *Canadian Journal of Forest Research*, *24*(6), 1260–1265. https://doi.org/https://doi.org/10.1139/x94-165

IBM Corp. (n.d.). *Overview of mathematical programming*. IBM® Decision Optimization CPLEX® Modeling for Python (DOcplex) V2.14 Documentation. Retrieved May 12, 2020, from http://ibmdecisionoptimization.github.io/docplex-doc/mp.html#cplex-algorithms

IBM Corp. (2017). *IBM ILOG CPLEX Optimization Studio CPLEX User's Manual*.

Johnson, K. N., & Scheurman, H. L. (1977). Techniques for Prescribing Optimal Timber Harvest and Investment Under Different Objectives—Discussion and Synthesis. *Forest*

Science, *23*(suppl_1), a0001-z0001. https://doi.org/10.1093/forestscience/23.s1.a0001

Joro, T., Korhonen, P., & Wallenius, J. (1998). Structural Comparison of Data Envelopment Analysis and Multiple Objective Linear Programming. *Management Science*, *44*(7), 962–970. https://doi.org/10.1287/mnsc.44.7.962

Kantorovich, L. (1939). Mathematical methods of organizing and planning production. *Management Sci.*, *6*(4), 366–422.

Keleș, S., & Bașkent, E. Z. (2011). A computer-based optimization model for multiple use forest management planning: a case study from Turkey. *Scientia Forestalis*, *39*(89), 87–95.

Keleş, S., Yolasiğmaz, H. A., & Başkent, E. Z. (2007). Long-term modelling and analyzing of some important forest ecosystem values with linear programming. *Fresenius Environmental Bulletin*, *16*(8), 963–972.

Kent, B., Bare, B. B., Field, R. C., & Bradley, G. A. (1991). Natural Resource Land Management Planning Using Large-Scale Linear Programs: The USDA Forest Service Experience with FORPLAN. *Operations Research*, *39*(1), 13–27. https://doi.org/10.1287/opre.39.1.13

Kidd, W. E., Thompson, E. F., & Hoepner, P. H. (1966). Forest Regulation By Linear Programming — A Case Study. *Journal of Forestry*, *64*(9), 611–613.

Koltai, T., & Tatay, V. (2011). A practical approach to sensitivity analysis in linear programming under degeneracy for management decision making. *International Journal of Production Economics*, *131*(1), 392–398. https://doi.org/10.1016/j.ijpe.2010.04.037

Koltai, T., & Terlaky, T. (2000). Difference between the managerial and mathematical interpretation of sensitivity analysis results in linear programming. *International Journal of Production Economics*, *65*(3), 257–274. https://doi.org/10.1016/S0925-5273(99)00036-5

Leavengood, J. R. and S. (2000). Using Duality and Sensitivity Analysis to Interpret Linear Programming Solutions. *Performance Excellence in the Wood Products Industry. Oregon State University*, *April*.

Lee, T. (2015). *Understanding advantages of optimization in general business operations*. *916*, 1–3. http://www.kisters.net/docs/Optimization-vs-Simulation.pdf

LINDO Systems. (2017). *LINGO The Modeling Language and Optimizer*. LINDO Systems Inc.

Lotov, A. V., Bushenkov, V., & Kamenev, G. K. (2004). *Interactive decision maps: Approximation and visualization of Pareto frontier*. Kluwer Academic Publishers.

Marques, S., Marto, M., Bushenkov, V., McDill, M., & Borges, J. G. (2017). Addressing wildfire risk in forest management planning with multiple criteria decision making methods. *Sustainability (Switzerland)*, *9*(2), 1–17. https://doi.org/10.3390/su9020298

Martins, H., & Borges, J. G. (2007). Addressing collaborative planning methods and tools in forest management. *Forest Ecology and Management*, *248*(1–2), 107–118. https://doi.org/10.1016/j.foreco.2007.02.039

McDill, M. E. (1999). *Forest resource management*. Penn State Univ.

McDill, M. E. (2014). An Overview of Forest Management Planning and Information Management. In J. G. Borges, L. Diaz-Balteiro, M. E. McDill, & L. C. E. Rodriguez (Eds.), *The Management of Industrial Forest Plantations: Theoretical Foundations and Applications,*. Springer. https://doi.org/10.1007/978-94-017-8899-1_5

Mendoza, G. A., & Martins, H. (2006). Multi-criteria decision analysis in natural resource management: A critical review of methods and new modelling paradigms. *Forest Ecology and Management*, *230*(1), 1–22. https://doi.org/https://doi.org/10.1016/j.foreco.2006.03.023

Mendoza, G. A., & Vanclay, J. K. (2008). Trends in forestry modelling. *CAB Reviews: Perspectives in Agriculture, Veterinary Science, Nutrition and Natural Resources*, *3*(April 2008). https://doi.org/10.1079/PAVSNNR20083010

Mohammadi, Z., Limaei, S. M., & Shahraji, T. R. (2017). Linear programming approach for optimal forest plantation. *Journal of Forestry Research*, *28*(2), 299–307. https://doi.org/10.1007/s11676-016-0318-y

Nieuwenhuis, M., & Biber, P. (2018). Milestone 11 - Projections with current FMMs per case study. In *University College Dublin—Technical University of Munich*.

Nijnik, M., Oskam, A., & Nijnik, A. (2012). Afforestation for the Provision of Multiple Ecosystem Services: A Ukrainian Case Study. *International Journal of Forestry Research*, *2012*, 295414. https://doi.org/10.1155/2012/295414

Öhman, K., & Eriksson, L. O. (2002). Allowing for spatial consideration in long-term forest planning by linking linear programming with simulated annealing. *Forest Ecology and Management*, *161*(1–3), 221–230. https://doi.org/10.1016/S0378-1127(01)00487-X

PHP Documentation Group. (2020). *PHP Manual*.

Pretzsch, H. (2009). Forest dynamics, growth, and yield. In *Forest dynamics, growth and yield* (pp. 1–39). Springer.

Sabogal, C., Guariguata, M. R., Broadhead, J., Lescuyer, G., Savilaakso, S., Essoungou, J. N., And, & Sist, P. (2014). Multiple-use forest management in the humid tropics Opportunities and challenges for sustainable forest management Opportunities and challenges for sustainable forest management. In *FAO Forestry Paper No. 173*. https://doi.org/10.1515/9781400830459.1

Sessions, J., Bettinger, P., & Murphy, G. (2007). *Heuristics in Forest Planning BT - Handbook Of Operations Research In Natural Resources* (A. Weintraub, C. Romero, T. Bjørndal, R. Epstein, & J. Miranda (eds.); pp. 431–448). Springer US. https://doi.org/10.1007/978-0-387-71815-6_23

The PostgreSQL Global Development Group. (2020). *PostgreSQL 9.6.18 Documentation*.

Tomé, M., & Faias, S. (2011). *Report describing the regional simulators and the European* *simulator. EFI Technical Report 69.*

Twery, M. J. (2014). Environmental modelling: finding simplicity in complexity. *Choice Reviews Online*, *51*(05), 51-2682-51–2682. https://doi.org/10.5860/choice.51-2682

Zhou, W., & Gong, P. (2004). Economic effects of environmental concerns in forest management: an analysis of the cost of achieving environmental goals. *Journal of Forest Economics*, *10*(2), 97–113. https://doi.org/https://doi.org/10.1016/j.jfe.2004.05.003